Non-Stationary Corona around Multi-Point System in Atmospheric Electric Field: Discharge Current and Vertical Electric Field Profile

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ABSTRACT: The properties of a non-stationary glow corona maintained near the tips of a multi-point ground system in a time-varying thundercloud electric field have been studied numerically. The discharge was simulated from a system of identical vertical conductive electrodes that is a model of the earth's surface extremities coronating under a thundercloud. The effect of system geometry and dimensions on the discharge properties and on vertical electric field profile above the coronating system was investigated. Conditions were determined under which the corona properties of a multi-point system are similar to the properties of a plane surface that emits ions into the atmosphere. The obtained results were used to estimate the temporal evolution of corona current density and corona space charge emitted during thunderstorms from the earth's surface covered with dense vegetation.

INTRODUCTION

Corona discharges developed from the earth's surface extremities (the tips of trees, bushes, leaves, grass and other sharp objects) under a thundercloud leads to the space charge injection into the atmosphere and make a contribution to the global electric circle. In addition, the corona space charge layer affects the local electric field at ground level and is practically important for lightning protection.

Laboratory studies of a corona discharge cannot be directly extended to thunderstorm conditions because a discharge occurring near grounded objects in a time-varying atmospheric electric field is non-stationary and the corona current depends on the manner in which the ambient field evolves in time, rather than on its instantaneous values. The reason is that, in this case, the space charge front has no time to bridge the gap and to reach the thundercloud, whereas the corona space charge reaches usually the opposite electrode on a laboratory scale.

The properties of a corona discharge developed from a solitary grounded hemispherically-tipped rod in a thundercloud electric field was considered analytically and numerically [Bazelyan and Raizer 2000; Aleksandrov et al. 2001; Bazelyan et al. 2008] on the basis of a simple 1D approximation. It was shown that the corona current varies in time as $i_{cor}(t) \sim t^{(3k-1)/2}\mu^{1/2}$, when the cloud electric field varies as $E_0(t) \sim t^k$, $k > -1$.

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Here, μ is the ion mobility. In this case, the discharge current is constant only when the electric field rises in time. In a steady electric field (k = 0), the current decreases with time. The effect of ion mobility on the current is smaller than this effect for laboratory gaps when $i_{cor}(t) \sim \mu$.

Recent time-consuming numerical 2D simulations for a solitary grounded rod [Becerra 2013] and for a long horizontal grounded wire [Mokrov et al. 2013] supported the use of a much simpler 1D calculations for a qualitative analysis when the focus is on the processes in the vicinity of the coronating surface or when computational time is limited. This simplification is especially important when considering the properties of a corona developed from a grounded multi-point system with a complicated geometry.

In a thundercloud electric field, the corona current even from an extremely high solitary electrode does not exceed 1 mA that is not important from the standpoint of the global electric circuit. Multi-point ground coronating systems (forest, bushes, grass and urban areas) make much larger contribution to the total current from the earth's surface. In this case, the local electric field near a given coronating point is affected not only by the space charges developed from this point, but by the space charges emitted by others corona sources as well. Numerical simulation of a corona discharge from a multipoint system is much more complicated than that from a solitary electrode since it is necessary to consider interaction between coronating points and individual corona space charge layers.

In this work, we extended the 1D approach developed in [Bazelyan and Raizer 2000; Aleksandrov et al. 2001; Bazelyan et al. 2008] for a solitary grounded electrode to a multi-electrode system. The properties of a non-stationary (transient) corona initiated and developed from a model multi-point ground system in a thundercloud electric field were numerically studied for different geometrical parameters of the system (see also [Bazelyan et al. 2014a]). A simplified method to determine the corona current density and injected corona space charge under real conditions was suggested. The evolution in time of vertical electric field profiles in the space charge layer above a multi-point system was also considered (see also [Bazelyan et al. 2014b]).

CORONA INITIATION FROM MULTI-POINT SYSTEM IN EXTERNAL ELECTRIC FIELD

In this work, the model of a corona discharge around a solitary electrode (see [Bazelyan and Raizer 2000; Aleksandrov et al. 2001; Bazelyan et al. 2008]) was generalized to study the discharge from a multi-point system. We considered a system of vertical grounded hemispherically-tipped electrodes under practically important conditions when the electrode height h is much higher than the curvature radius of the electrode top, r_0, and the distance between adjacent electrodes, D, is comparable with h. Electrodes were uniformly distributed over concentric circles with the radii $r_k = kD$ (k = 1,2…) around a given electrode (see figure 1). It was assumed that $6k$ electrodes are located on the k-th circle and that the total number of electrodes is such large that almost every coronating point is surrounded by numerous similar points. This allowed calculation of discharge properties only for the central electrode under the assumption that the discharge properties for other electrodes are similar. (Here, the peculiarities of the corona discharge near the electrodes at the outer boundary of the system were neglected.)

The same approximation was used to calculate the corona onset atmospheric electric field, E_{0cor}, at which the local electric field near the electrode tips reaches the corona onset field, E_{cor}, and corona is ignited. The value of E_{cor} was determined from the empirical formula suggested by Bazelyan et al. (2007).

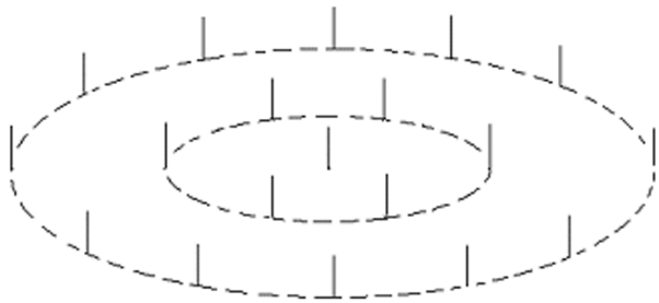

Fig. 1. The distribution of electrodes over the ground surface.

A quantitative relation between E_{0cor} and E_{cor} for a given multi-electrode system can be calculated using available electrostatic numerical methods. Figure 2 shows the threshold atmospheric electric field E_{0cor} calculated with the charge simulation method [Malik 1989] for a system of grounded spherical electrodes as a function of the number of circles with surrounding electrodes. The value of E_{0cor} increases with the number of surrounding electrodes and is affected even by electrodes located at a distance of 100 m. This is explained by the fact that the number of surrounding electrodes distributed over a given circle increases with the circle radius; that is, the distant circles contain much more surrounding electrodes and each of these electrodes makes a contribution into the potential of the central electrode. The value of E_{0cor} even for the multipoint system with closely packed electrodes (D/h =0.1) is only double that E_{0cor} for a solitary electrode (N = 0).

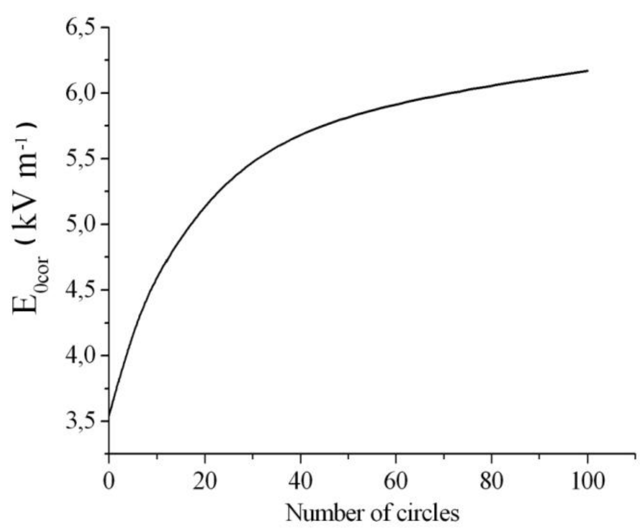

Fig. 2. The threshold ambient electric field required for corona initiation in a multi-point system as a function of the number of circles with surrounding electrodes. The calculation was made for h = 10 m, D = 1m and r_0 = 1 cm.

CALCULATED MODEL OF CORONA DISCHARGE

A physical approach to simulating a non-stationary, streamer-free, glow corona of positive polarity initiated from grounded electrodes in an atmospheric electric field and algorithms applicable to the simplest electrode geometries has been given in detail elsewhere [Aleksandrov et al. 2002]. In this model, the ionization layer in the immediate vicinity of the electrode tip was not considered because its thickness is much smaller than the radius of curvature of the tip. Here, the corona-producing surface was assumed to be an emitter of ions and the boundary condition for electric field was reduced to a condition widely used to determine the current-voltage characteristic of a stationary glow corona in long gas gaps [Raizer 1991], namely, that electric field at a coronating surface is equal to the onset corona field, E_{cor}. For a hemispherically-tipped rod with radius r_0, the boundary condition was reduced to

$$E(r_0) = E_{\mathrm{cor}}. \qquad (1)$$

For the sake of definiteness, we assumed that an external electric field was produced by a time-varying thundercloud negative charge. The expansion of the corona space charge layers was described by the electrostatic equation for electric field

$$\mathrm{div}\, \boldsymbol{E}(\boldsymbol{r}) = \rho/\varepsilon_0 \qquad (2)$$

and continuity equations for ions

$$\frac{\partial n_j}{\partial t} + div\left(n_j \mu_j E\right) = S_j\,, \qquad (3)$$

where $\rho = e\Sigma n_{\mathrm{j}}$ is the space charge density, e is the charge of a singly charged ion, n_{j} and μ_{j} are the number density and mobility of ions of species j, respectively, and S_{j} is a source term describing ion-molecule reactions that affect the ion composition and, hence, the ion transport. The potential φ introduced as $\boldsymbol{E} = -\nabla\varphi$ was assumed to tend to zero at the grounded plane and at grounded electrodes, whereas, away from them and from the ion “cloud”, the electric field tended to the undisturbed external electric field, $E_0(t)$.

Electric field above every coronating electrode was calculated taking into account not only the corona space charge emitted by this electrode, but the charges emitted by other electrodes as well. The effect of these charges was considered approximately, assuming that they are point charges.

NUMERICAL SIMULATION OF CORONA CURRENT AND INJECTED SPACE CHARGE

Our numerical simulation showed the following peculiarities of a corona discharge from a multi-point system.

Corona current decreases with increasing the number of coronating sources (see figure 3), whereas the rate of decrease of the corona current at E_0 = const increases in this case. The temporal evolution of the corona current, $i_{\mathrm{cor}}(t)$, is easy to analyze in figure 4 where the values are normalized to the peak corona currents, i_{max}.

In a multi-point system with a few thousand of electrodes, where the corona current is stabilized in a linearly rising thundercloud electric field, the value of the stabilized current, $i_{cor\,max}$ is almost independent of the electrode height (see figure 5) and depends strongly on the distance between electrodes, D (see figure 6). It follows from the data that $i_{cor\,max} \sim D^2$.

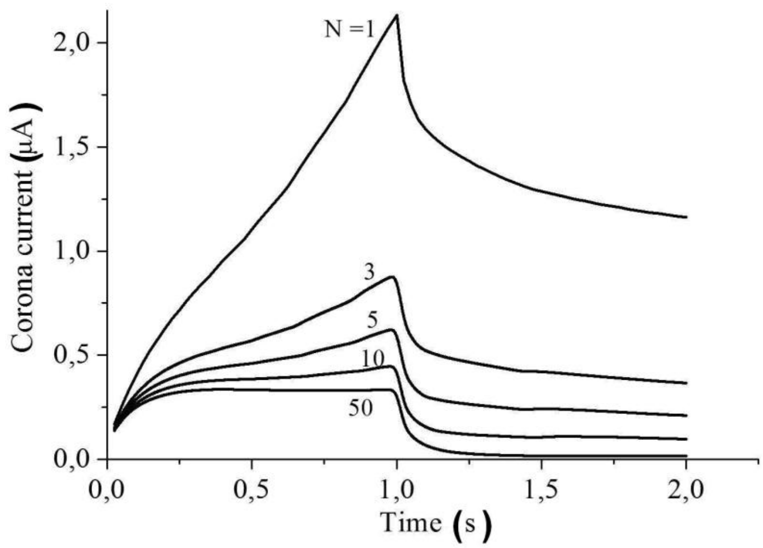

Fig. 3. The evolution in time of the corona current from the top of the central electrode in a multi-rod system with rods for $h = D = 1$ m and $r_0 = 10^{-1}$ cm. The external electric field rises linearly from zero to E_{0m} at $t < t_m$ and is equal to E_{0m} at $t > t_m$, where $E_{0m} = 40$ kV m^{-1} and $t_m = 1$ s.

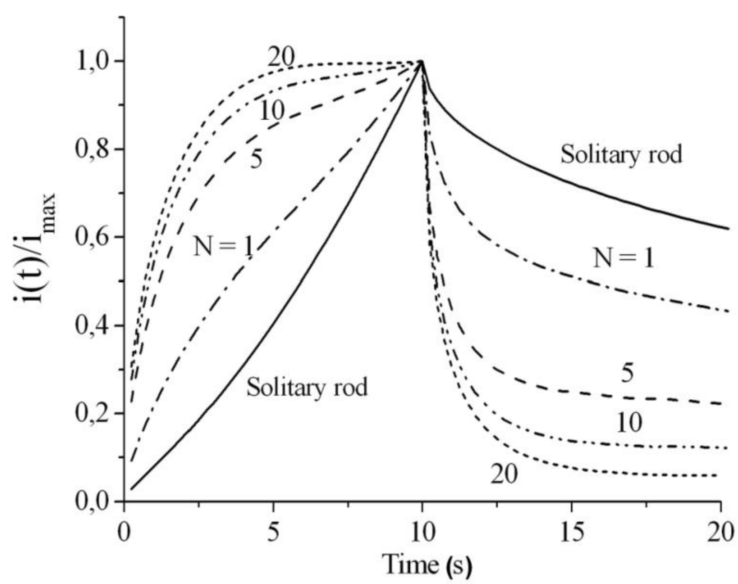

Fig. 4. The evolution in time of the corona current from the top of the central electrode in a multi-rod system with rods for $h = D = 10$ m and $r_0 = 1$ cm. The external electric field rises linearly from zero to E_{0m} at $t < t_m$ and is equal to E_{0m} at $t > t_m$, where $E_{0m} = 20$ kV m^{-1} and $t_m = 10$ s.

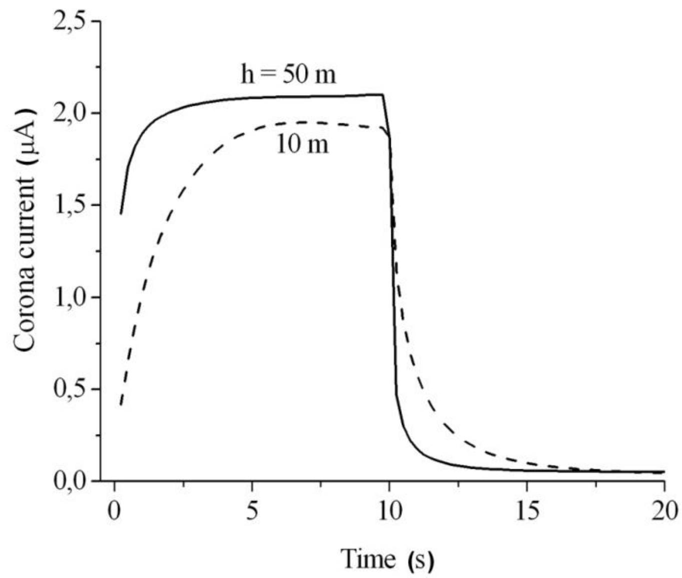

Fig. 5. The evolution in time of the corona current from the top of the central rod in a multi-point system with rods of height h = 10 and 50 m. The number of circles with surrounding rods is N = 50. Other conditions are similar to those in figure 4.

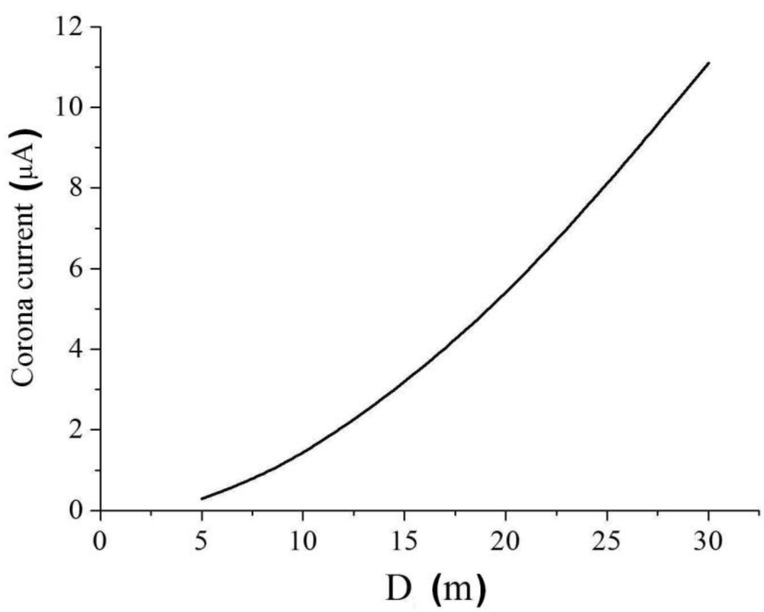

Fig. 6. The value of the stabilized corona current from the top of the central rod in a multi-point system with rods of height h = 10 m as a function of the distance between electrodes. The number of circles with surrounding rods is N = 50. The external electric field rises linearly from zero to 40 kV m^{-1} for 30 s.

The time it takes to saturate the corona current for a multi-point system in a linearly rising external electric field also depends on the distance between electrodes; this dependence is close to a linear one (see figure 7).

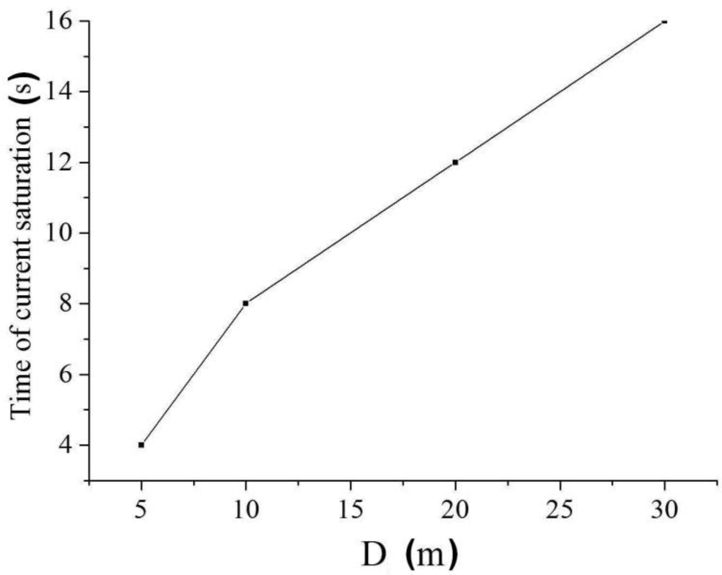

Fig. 7. The time it takes to saturate the corona current for a multi-point system in a linearly rising external electric field as a function of the distance between electrodes. Conditions are similar to those in figure 6.

Analysis of our calculations shows that the properties of a multi-point coronating system asymptotically tend to those of a prefect emitting plane with the surface electric field that is equal to the corona onset atmospheric electric field E_{0cor} [Bazelyan et al. 2008]. Stabilization of the surface electric field is due to ion emission. Indeed, the plane space charge layer and its image in the conducting ground form a double electrostatic layer; that is, the electric field is equal to $E_0(t)$ at the upper boundary of the layer and to E_{0cor} at the ground surface. In this case, it follows from the Poisson equation (the Gauss theorem) that, to stabilize the surface electric field at the level E_{0cor}, the corona space charge injected into the atmosphere per unit area must be [Bazelyan et al. 2008]

$$q(t) = \varepsilon_0 [E_0(t) - E_{0cor}]. \qquad (4)$$

Then, the corona current density is expressed as

$$j_{cor}(t) = \varepsilon_0 \frac{dq}{dt} = \varepsilon_0 \frac{dE_0(t)}{dt}. \qquad (5)$$

It follows from (5) that in the asymptotic limit the corona current density depends only on the rate of rise of the external electric field, $E_0(t)$. In particular, the current must be constant for a linearly rising electric field and must tend to zero for a constant electric field. It is precisely this manner of the temporal evolution of the corona current is obtained from our calculations for multi-point systems when the number of coronating electrodes is sufficiently large. The current through one electrode in multi-point systems studied is obtained by taking the product of j_{cor} and the area per one electrode in the system, $S = \pi D^2N^2/n_{el}$, where N is the number of circles covered with electrodes and n_{el} is the total number of electrodes in the system. Then, we have

$$i_{cor}(t) \approx \frac{\pi D^2 N^2}{n_{el}} j_{cor}(t) = \frac{\pi \varepsilon_0 D^2 N^2}{n_{el}} \frac{dE_0(t)}{dt}. \quad (6)$$

From (6), $i_{cor\ max} \sim D^2$, in agreement with our calculations (see figure 6). Moreover, there is good quantitative agreement between equation (6) and our calculated results. For instance, it follows from the results shown in figure 6 that $i_{cor\ max}$ = 5.04 μA for the system with D = 20 m, whereas the current obtained from (6) under the same conditions is 4.85 μA. Here, the difference is less than 5%.

The calculated corona current actively increases in time due to the development of individual corona space charges from their sources until a united corona space charge layer is formed. In the end, individual space charges unite into one plane corona space charge layer (see figure 8) and then the model of emitting plane (equations (4) and (5)) becomes adequate.

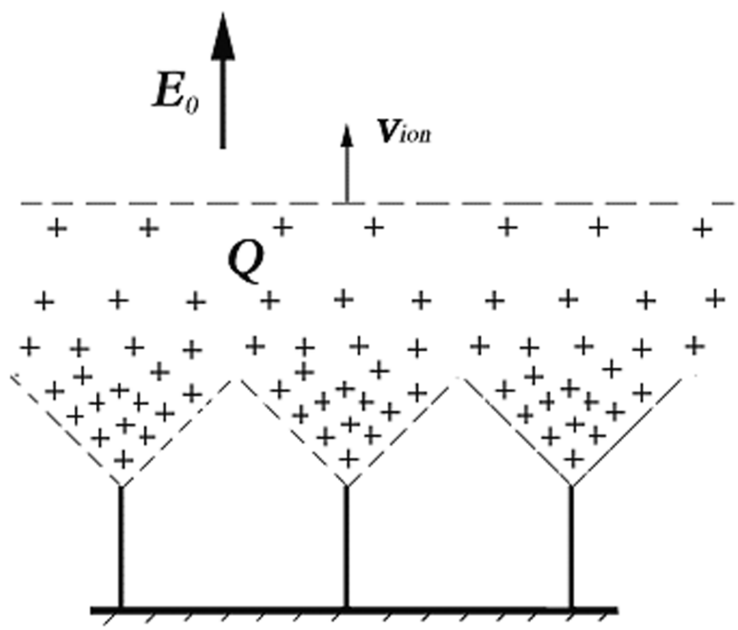

Fig. 8. A schematic diagram of the space charge layer formed above a ground multi-points system in an atmospheric electric field E_0.

According to our calculations, the duration of the phase of active current growth in a multi-point system corresponds to the time it takes for the fronts of the individual space charge “clouds” to develop from the coronating sources until the formation of a united space charge layer. This time can be estimated as the time when the radius of the front of an individual space charge “cloud”, R_f, reaches D/2 (see figure 9).

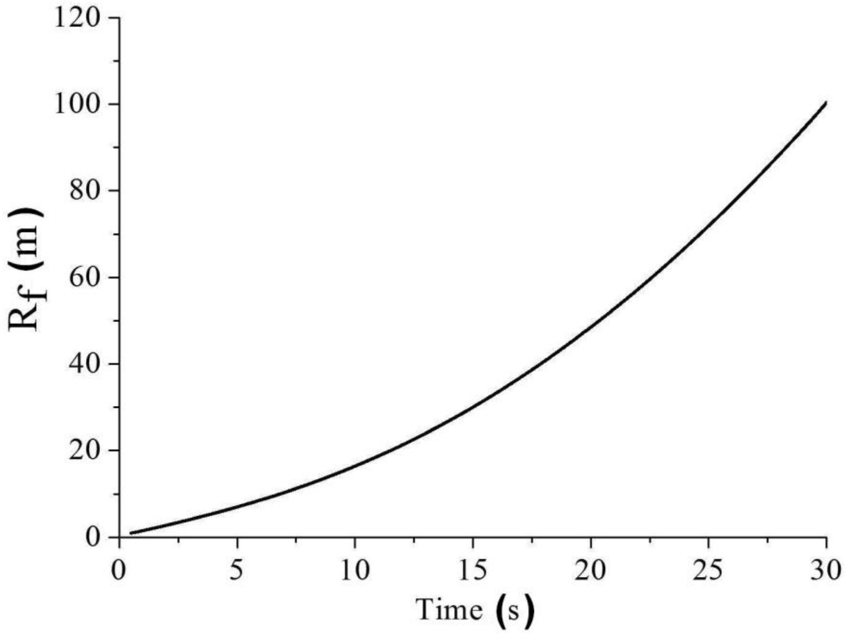

Fig. 9. The evolution in time of the radius of the front of an individual space charge “cloud” developed from a central electrode in a multi-point system with D = 20 m. Conditions are similar to those in figure 6.

It may be concluded that, to calculate the corona current emitted from a unit area of the ground surface during thunderstorms, there is no need to consider geometry of coronating extremities on the ground surface. With a good accuracy, current density could be estimated from the rate of rise of an undisturbed thundercloud electric field using equation (5). The corona space charge emitted from a unit area of the ground surface can be estimated in a similar way. From (4), this charge depends on the geometry properties of a coronation system only indirectly, via the corona onset atmospheric electric field, E_{0cor}. Under most practically important thunderstorm conditions, we have $E_0 >> E_{0\mathrm{cor}}$. In this case, the value of q turns out to be independent of the system parameters and is equal to

$$q_{max} \approx \varepsilon_0 E_{0max} , \qquad (7)$$

where E_{0max} is the peak thunderstorm electric field. For instance, we have $q_{max} \approx 0.53$ μC m^{-2} for E_{0max} = 60 kV m^{-1} [Soula and Chauzy 1991].

ELECTRIC FIELD PROFILES ABOVE MULTI-POINT CORONATING SYSTEM

Our calculations showed that corona properties for a multi-point system are controlled by an undisturbed thundercloud electric field, $E_0(t)$. Its direct measurement is not easy to make because of the effect of corona space charge layer. The local electric field near coronating sources is stabilized at the level of the corona onset electric field. Electric field in the corona space charge layer is lower than E_0 due to this charge and, only outside of the layer (outside of the double electrostatic plane layer), a thundercloud electric field is not disturbed.

In an 1D approximation, electric field profiles above an emitting plane can be exactly found from equations (2) and (3) in an analytical way [Bazelyan et al. 2014b]. Figure 10 shows the temporal evolution of the electric field at different altitudes in this case when the thundercloud electric field rises linearly up to 60 kV m^{-1} for 30 s and then is kept constant. Electric field at any altitude is equal to the thundercloud electric field until the front of the space charge layer reaches this altitude. Then, the local electric field, $E(t)$, is stabilized. Stabilization is obtained only for a linearly rising thundercloud field, $E_0(t) \sim t$. In the general case the local electric field inside the corona space charge layer increases in time for $d^2E_0/dt^2 > 0$ and decreases in time at $d^2E_0/dt^2 < 0$. This means that a sensor, being placed inside the corona space charge layer, registers a local electric filed that not only can differ quantitatively from the undisturbed thundercloud electric field, but can have even opposite temporal tendency as well. This is demonstrated in figure 11 that shows the temporal evolution of the electric field at different altitudes above an emitting plane when the thundercloud electric field $E_0(t)$ rises in time in a relaxation manner,

$$E_0(t) = E_{0\max}\left(1 - e^{-t/\tau}\right). \qquad (8)$$

Here, we have $d^2E_0/dt^2 < 0$ and the local electric field inside the space charge layer decreases in time although $dE_0/dt > 0$.

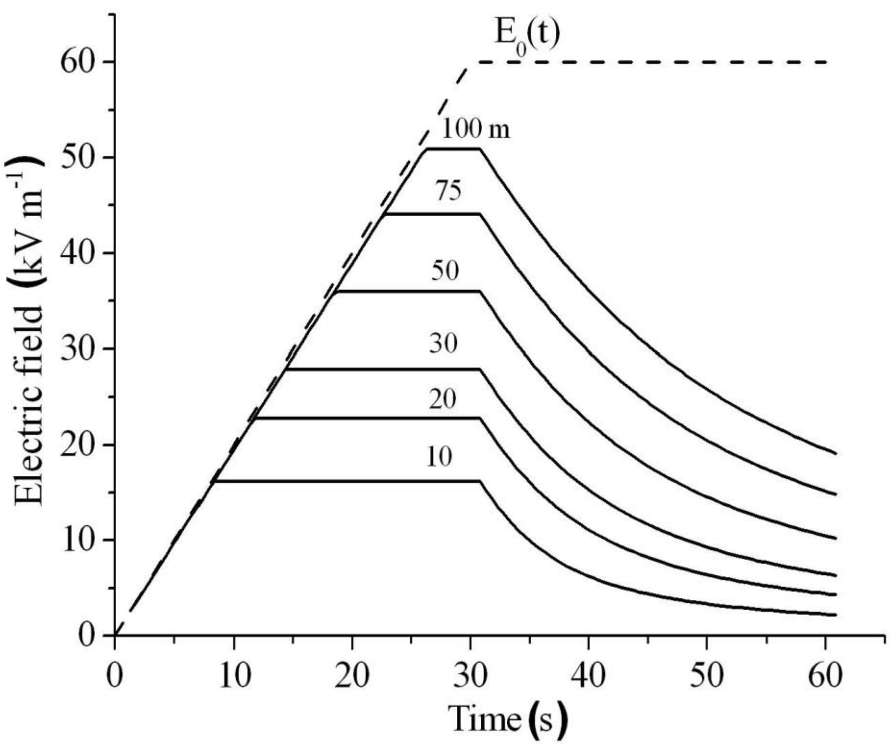

Fig. 10. The evolution in time of the electric field at different altitudes above an emitting plane at E_{0cor} =1.65 kV m^{-1}. The dashed curve corresponds to the thundercloud electric field that rises linearly in time up to $E_{0\max}$ = 60 kV m^{-1} for t_m = 30 s and then is kept constant.

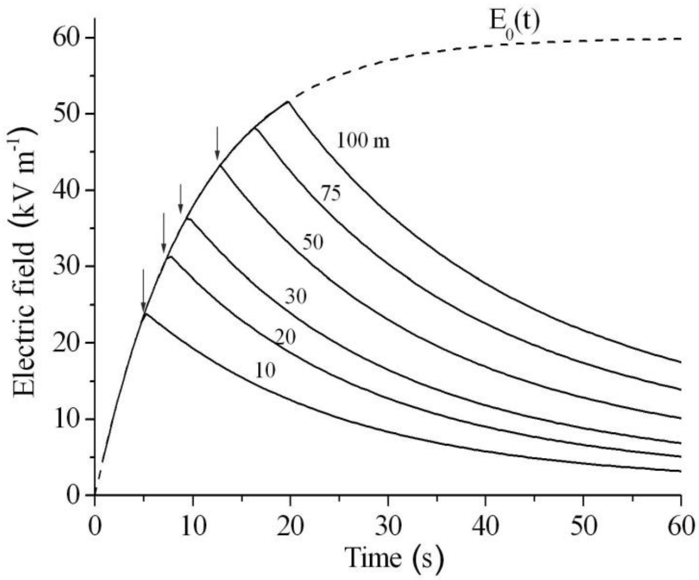

Fig. 11. The evolution in time of the electric field at different altitudes above an emitting plane at E_{0cor} =1.65 kV m^{-1}. The dashed curve corresponds to the thundercloud electric field that varies as (8) at $E_{0\,max}$ = 60 kV m^{-1} and τ = 10 s. The arrows indicate the instants at which the top boundary of the space charge layer reaches given altitudes.

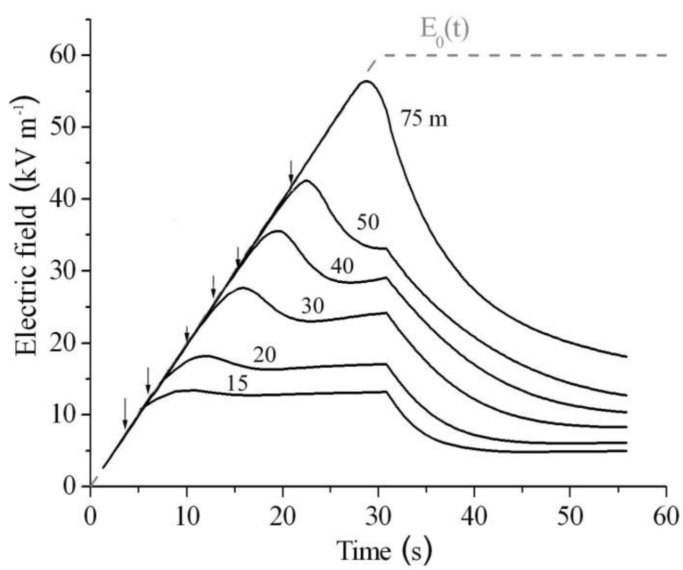

Fig. 12. The evolution in time of the electric field at different altitudes above the central rod in a multi-point system with rods of height h = 10 m and radius r_0 = 2 cm. The distance between rods is D = 10 m. The number of circles with surrounding rods is N = 100. The altitude is reckoned from the ground. The dashed curve corresponds to the thundercloud electric field that rises linearly in time up to $E_{0\,max}$ = 60 kV m^{-1} for t_m = 30 s and then is kept constant. The arrows indicate the instants at which the top boundary of the space charge layer reaches given altitudes.

Stabilization of the thundercloud electric field at $t > t_m$ leads to a collapse of the corona current. In this case, the corona space charge layer ascends and expands because the top front of the layer moves with a velocity $v_f = \mu E_{0max}$, whereas the velocity of the bottom boundary of the layer is lower, $v_b = \mu E_{0cor}$. The total electric field behind the top front of the layer decreases in time and tends to E_{0cor}, the electric field at the bottom boundary of the layer.

Figure 12 shows the temporal evolution of the electric field inside the space charge layer above a model multi-point coronating system. The distance between the rods in the system was equal to the rod height. Similarity between the data in figures 12 and 10 is close. In both cases, the total electric field $E(t)$ (i) is close to the undisturbed thundercloud electric field, $E_0(t)$, at altitudes above the space charge front, (ii) is stabilized (although with some delay) inside the space charge layer at $E_0 = At$ and (iii) sharply decreases at E_0 = const. Our calculations show that the vertical electric field profile above a multi-point coronating system tends to the electric field profile above a plane surface emitting ions as the number of electrodes in the system increases.

CONCLUSIONS

The developed computer model allows quantitative estimation of the properties of a non-stationary glow corona in the system of grounded hemispherically-tipped electrodes in a thundercloud electric field E_0. The properties of the multi-point coronating system asymptotically tend to those of a prefect emitting plane with the surface electric field that is equal to the corona onset atmospheric electric field E_{0cor}. The field E_{0cor} is controlled by the dimensions of the individual electrodes and by the distance between them. It is shown that the model of an emitting plane is valid when the individual space charge layers from different coronating points reach each other and form a unite plane layer. The time it takes for the formation of the united layer depends on the distance between coronating electrodes.

In the asymptotic approximation, the corona current density is equal to $\varepsilon_0 dE_0/dt$. In this case, the current through each coronating point is independent of the dimensions of the electrodes and depends only on the distance between them. The total corona space charge injected into the atmosphere per unit area of a multi-point system tends asymptotically to the expression $q = \varepsilon_0(E_0 - E_{0\mathrm{cor}})$ and depends on the geometrical parameters of the electrodes only indirectly, through the corona onset atmospheric electric field $E_{0\mathrm{cor}}$. Under practically important thunderstorm conditions, it is generally follows from field observations that $E_0 >> E_{0\mathrm{cor}}$. In this case, the value of q turns out to be independent of the system parameters.

The vertical electric field profile above a multi-point coronating system tends to the electric field profile above a plane emitting surface as the number of electrodes in the system increases. As a result, the electric field distribution tends to be independent of the height of coronating points, whereas the spacing between the electrodes affects only the time it takes to stabilize the electric field profile.

Electric field at a given altitude above the ground coronating surface in a thundercloud electric field is equal to this field until the space charge layer reaches this altitude. The evolution in time of the electric field E measured in the space charge layer depends on the rate of change of the thundercloud electric field E_0. The field E (i) undergoes a stabilization when the value of E_0 rises linearly in time, (ii) increases in time at $d^2E_0/dt^2 > 0$ and decreases in time at $d^2E_0/dt^2 < 0$. Consequently, simultaneous measurements of electric field at various levels could produce not only various results, but radically different evolutions in time as well.

ACKNOWLEDGMENTS

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